ANDHRA UNIVERSITY

SCHOOL OF DISTANCE EDUCATION

ASSIGNMENT QUESTION PAPER

M.A. / M.Sc (Previous) Mathematics - Supplementary

Paper - I ALGEBRA

Answer ALL Questions

All Questions carry equal marks

Section - A **(4 x 4 =16 Marks)**

1. (a) Prove that a homomorphism $\phi : G \to H$ is injective if and only if $ker\ \phi = \{e\}$.

   (b) State and prove Cayley's theorem

2. (a) Prove that alternating group A_n is simple if $n > 4$. Consequently S_n is not solvable if $n > 4$.

   (b) State and prove Cauchy's theorem for abelian group.

3. (a) Let $f : R \to S$ be a homomorphism of a ring R into a ring S. Then prove that $ker\ f = (0)$ if and only if f is 1-1.

   (b) If R is a commutative ring, then prove that an ideal P in R is prime if and only if $ab \in P,\ a \in R,\ b \in R,$ implies $a \in P$ or $b \in P$.

4. (a) Let $f(x) \in \mathbb{Z}[x]$ be prime. Then prove that $f(x)$ is reducible over Q if and only if $f(x)$ is reducible over $\mathbb{Z}$.

   (b) Let E and F be fields and let $\sigma : F \to E$ be an embedding of F into E. Then prove that $\exists$ a field K such that F is a subfield of K and σ can be extended to an isomorphism of K onto E.

Section - B **(4 x 1 = 4)**

5. Answer all the following :

(a) Let G be a group and $a, b \in G$ such that $ab = ba$. If $o(a) = m,\ o(b) = n$ and $(m,n) = 1$ then prove that $o(a,b) = mn$.

(b) Express the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 5 & 1 & 6 & 4 & 3 \end{pmatrix}$ as a product of disjoint cycles.

(c) Prove that the centre of a ring is a subring.

(d) Find the smallest extension of Q having a root of $x^2 + 4 \in Q[x]$.

ANDHRA UNIVERSITY
SCHOOL OF DISTANCE EDUCATION
ASSIGNMENT QUESTION PAPER
M.A. / M.Sc (Previous) Mathematics- Supplementary
Paper-II LINEAR ALGEBRA AND DIFFERENTIAL EQUATIONS

Answer ALL Questions
All Questions carry equal marks
Section - A **(4 x 4 =16 Marks)**

1. (a) Let T be a linear operator on an $n-$ dimensional vector space V. Then prove that the characteristic and minimal polynomials for T have the same roots, except for multiplicities.
   (b) State and prove Cayley - Hamilton theorem.
2. (a) If $y_1(x)$ and $y_2(x)$ are any two solutions of $y''+p(x)y'+Q(x)y=0$ on $[a,b]$, then prove that their Wronskian $W=W(y_1, y_2)$ is either identically zero or never zero on $[a,b]$.
   (b) Use method of variation of parameters, solve $y''+y=cosec$.
3. (a) If $W(t)$ is the Wronskian of the two solutions $x=x_1(t),\ y=y_1(t)$ and $x=x_2(t),\ y=y_2(t)$ of the homogeneous system $\frac{dx}{dt}=a_1(t)\,x+b_1(t)\,y;$ $\frac{dy}{dt}=a_2(t)\,x+b_2(t)\,y \to (1)$ are linearly independent on $[a,\ b]$, then prove that $x=c_1x_1(t)+c_2x_2(t);\ y=c_1y_1(t)+c_2y_2(t)$ is the general solution of system (1) on this interval.
   (b) Find the general solution of the system $\frac{dx}{dt}=-3x+4y;\ \frac{dy}{dt}=-2x+3y$.
4. (a) By the method Laplace transforms, find the solution of
   $y''-4y'+4y=0,\ y(0)=0\ and\ y'(0)=3.$
   (b) State and prove convolution theorem on Laplace transforms.

Section - B **(4 x 1 = 4)**

5. Answer all the following

(a) Let $T \in L(R), F = \mathbb{R}$ and matrix of T w.r.t. the standard basis is $\begin{bmatrix} 5 & 3 \\ -6 & -4 \end{bmatrix}$. Find the characteristic and minimal polynomials of T.

(b) Consider two functions $f(x) = x^3$ and $g(x) = x^2|x|$ on the interval $[-1, 1]$. Show that their Wronksian $W(f, g)$ vanishes identically.

(c) Find the general solution of the system

$$\begin{cases} \dfrac{dx}{dt} = x \\ \dfrac{dy}{dt} = y \end{cases}$$

(d) Find the inverse Laplace transforms of $\dfrac{12}{(p+3)^4}$.

ANDHRA UNIVERSITY
SCHOOL OF DISTANCE EDUCATION
ASSIGNMENT QUESTION PAPER
M.A. / M.Sc (Previous) Mathematics - Supplementary

Paper - III REAL ANALYSIS

Answer ALL Questions

All Questions carry equal marks

Section - A **(4 x 4 =16 Marks)**

1. (a) If $\{p_n\}$ is a sequence in a compact metric space X, then prove that some subsequence of $\{p_n\}$ converges to a point of X.

   (b) Let f be defined on $[a,b]$; if f has a local maximum at a point $x \in (a,b)$ and if $f'(x)$ exists, then prove that $f'(x)=0$.

2. (a) Assume α increases monotonically and $\alpha' \in R$ on $[a, b]$. Let f be a bounded real function on $[a, b]$. Then prove that $f \in R(\alpha)$ if and only if $f\alpha' \in R$. In that case, $\int_a^b f\, d\alpha = \int_a^b f(x)\alpha'(x)dx.$.

   (b) State and prove the fundamental theorem of calculus.

3. (a) Suppose $\{f_n\}$ is a sequence of functions, differentiable on $[a,\text{b}]$ and such that $\{f_n(x_0)\}$ converges for some point x_0 on $[a,\text{b}]$. If $\{f_n'\}$ converges uniformly on $[a,b]$, then prove that $\{f_n\}$ converges uniformly on $[a,\text{b}]$, to a function f *and* $f'(x)=\lim_{n\to\infty} f_n'(x)(a \le x \le b)$.

.

   (b) Suppose $\sum c_n$ converges. Put $f(x)=\sum_{n=0}^{\infty} c_n x^n$, $(-1<x<1)$. Then prove that $\lim_{x\to 1} f(x)=\sum_{n=0}^{\infty} c_n$.

4. (a) Suppose $\overline{f}$ maps on open set $E \subset \mathbb{R}^n$ into $\mathbb{R}^m$ *and* $\overline{f}$ is differentiable at a point $\overline{x} \in E.$ Then prove that the partial derivatives $(D_j f_i)(\overline{x})$ exist and

$$f'(\bar{x})e_j = \sum_{i=1}^{m} (D_j\, f_i)(\bar{x})\, y_i \;\; (1 \le j \le n).$$

(b) State and prove contraction principle.

Section - B **(4x1=4)**

5. Answer all the following.

(a) If $0 \le x < 1$, then prove that $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$.

(b) If $f \in R(\alpha)$ and $g \in R(\alpha)$ on $[a,b]$ then prove that $f+g \in R(\alpha)$ on $[a,b]$ and

$$\int_a^b (f+g)\, d\alpha = \int_a^b f\, d\alpha + \int_a^b g\, d\alpha.$$

(c) Prove that every uniformly convergent sequence of bounded functions is uniformly bounded.

(d) If $f(x,y) = \begin{cases} \frac{xy}{x^2+y^2} & for\ (x,y) \ne (0,0) \\ 0 & for\ (x,y) = (0,0) \end{cases}$ then prove that $(D_1 f)(x,y)$ and $(D_2 f)(x,y)$ exist at every point of $\mathbb{R}^2$, although f is not continuous at $(0,0)$.

ANDHRA UNIVERSITY

SCHOOL OF DISTANCE EDUCATION

ASSIGNMENT QUESTION PAPER

M.A. / M.Sc (Previous) Mathematics - Supplementary

Paper - IV TOPOLOGY

Answer ALL Questions

All Questions carry equal marks

Section - A **(4 x 4 =16 Marks)**

1. (a) Let X be a metric space then prove that a subset G of X is open $\Leftrightarrow$ it is a union of open spheres.

   (b) Let X and Y be metric spaces and f a mapping of X into Y. Then prove that f is continuous if and only if $f^{-1}(G)$ is open in X whenever G is open in Y.

2. (a) State and prove Lindelof 's theorem.

   (b) Prove that every sequentially compact metric space in totally bounded.

3. (a) Prove that every compact Hausdorff space is normal.

   (b) Show that a Hausdorff space is locally compact if and only if each of its points is an interior point of some compact space.

4. (a) State and prove real Stone - Weirstrass theorem.

   (b) Show that $C_o(X,\mathbb{R})$ and $C_o(X,\mathbb{C})$ are closed sub spaces of $C(X,\mathbb{R})$ and $C(X,\mathbb{C})$ respectively.

Section - B **(4 x 1 = 4)**

5. Answer all the following :

(a) Let X be an arbitrary non-empty set, and define d by $d(x,y)=\begin{cases}0 \; if \;\; x=y \\ 1 \; if \;\; x\neq y\end{cases}$. Then prove d is a metric on X.

(b) Show that a subspace of a topological space is itself a topological space.

(c) Show that any continuous image of a compact space is compact.

(d) If X is a locally compact Hausdorff space, then prove that $C_o(X,R)$ is a sublattice of $C(X,R)$.

ANDHRA UNIVERSITY
SCHOOL OF DISTANCE EDUCATION
ASSIGNMENT QUESTION PAPER
M.A. / M.Sc (Previous) Mathematics - Supplementary
Paper-V DISCRETE MATHEMATICS
Answer ALL Questions
All questions carry equal marks

Section - A **(4 x 4 =16 Marks)**

1. (a) Prove that a graph is bipartite if and only if it contains no odd cycles.
   (b) Let $G(V,E)$ be a graph with no isolated vertex. Then show that G has an Euler circuit if and only if G is connected and the degree of every vertex of G is even.
2. (a) Show that every distributive lattice is modular. Is the converse of this result true? Justify your claim.
   (b) Let B be a Boolean algebra. Prove that an ideal M in B is maximal if and only if for any $b \in B$ either $b \in M$ or $b' \in M$, but not both hold.
3. (a) Describe an automaton and semi automaton.
   (b) Explain by means of an example the concept of an automaton associated with a monoid $(S, \bullet)$. Show that there exists an automaton whose monoid is isomorphic to $(S, \bullet)$.
4. (a) State and prove the Hamming bound theorem.
   (b) Let C be an ideal $\neq \{0\}$ *of* V_n. Then prove that there exists a unique $g \in V_n$ with the following properties.
   (i) $g \mid x^n - 1$ *in* $F_q[x]$ (ii) $C = (g)$ (iii) g is monic.

Section - B **(4 x 1 = 4)**

5. Answer all the following :

(a) Show that a graph is a tree if and only if it has no cycles and $|E| = |V| - 1$.
(b) Determine the symbolic representation of the circuit given by

$$p = (x_1 + x_2 + x_3)(x' + x_2)(x_1x_3 + x_1' x_2)(x_2' + x_3)$$

(c) Define the group kernel of a monoid (S,o). Show that the group kernel G_5 is a group within (S,o).
(d) Show that a linear code $C \subseteq V_n$ is cyclic if and only if C is an ideal in V_n.